Mike's Problem (Or "Circling the Rectangle")

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Last summer I was approached by another teacher, Mike, with this problem: "I want to make a round table from a 4' x 8' sheet of plywood. How should I cut it to make the diameter of the table as large as possible?" The ideal solution, at least to a math teacher, would be to grind the entire sheet into sawdust - an "infinite" number of pieces - then glue it back together as a circle of diameter 2 * SQRT(48 x 96 / pi) ~ 76.6". While obviously impractical, this does at least set an upper bound on what is possible.

To make the problem tractable, let's limit things to one straight cut joining any two edges of the sheet. The two resulting pieces are to be glued edgewise, and a circle cut from the result. How would you make the cut and rejoin the pieces to maximize the diameter of the table? First, let me say this: I'm not sure! I will present a number of general approaches, but I'm open to improvements on my best result.

In all figures given, rectangle ABCD is the 4' x 8' sheet, segment EF is the location of the cut, O is the center of the circle, and *r* denotes radius. Other letters are introduced as needed, with a prime indicating the image of a point after rejoining the pieces.

Approach 1: Making a cut parallel to a long edge is doomed, but how about a cut parallel to a short edge (Figure 1)? The chord at the seam, E'F', is limited by the width of the sheet, 48". Applying the Pythagorean Theorem, we find that by making the cut 12" from, and parallel to, a short edge will produce a diameter of 60".

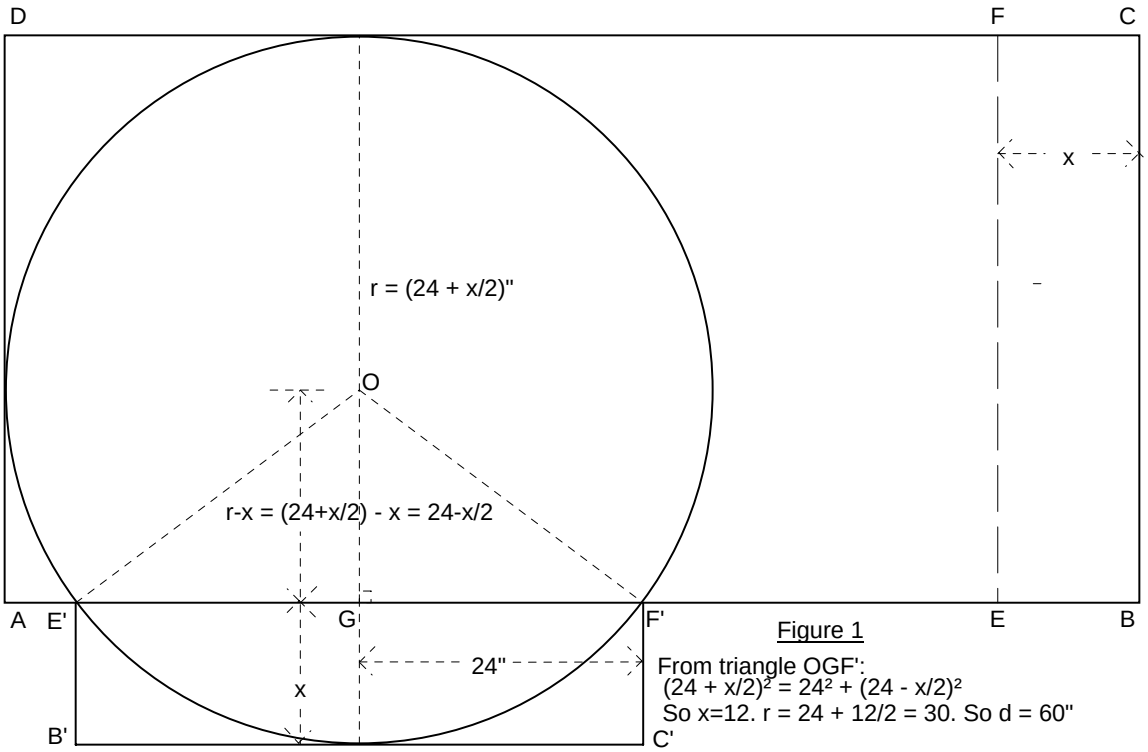

Figure 1

From triangle OGF':
$(24 + x/2)^2 = 24^2 + (24 - x/2)^2$
So x=12. r = 24 + 12/2 = 30. So d = 60"

***Approach 2*:** Make a symmetric cut joining two opposite edges (Figure 2). The best cut here also produces a 60" diameter. Rotating segment EF around point H, the center of the sheet, fails to produce improvement. (Try it!) When segment EF forms a diagonal of the sheet, overlapping segment BD, the resulting diameter is ~ 59.3".

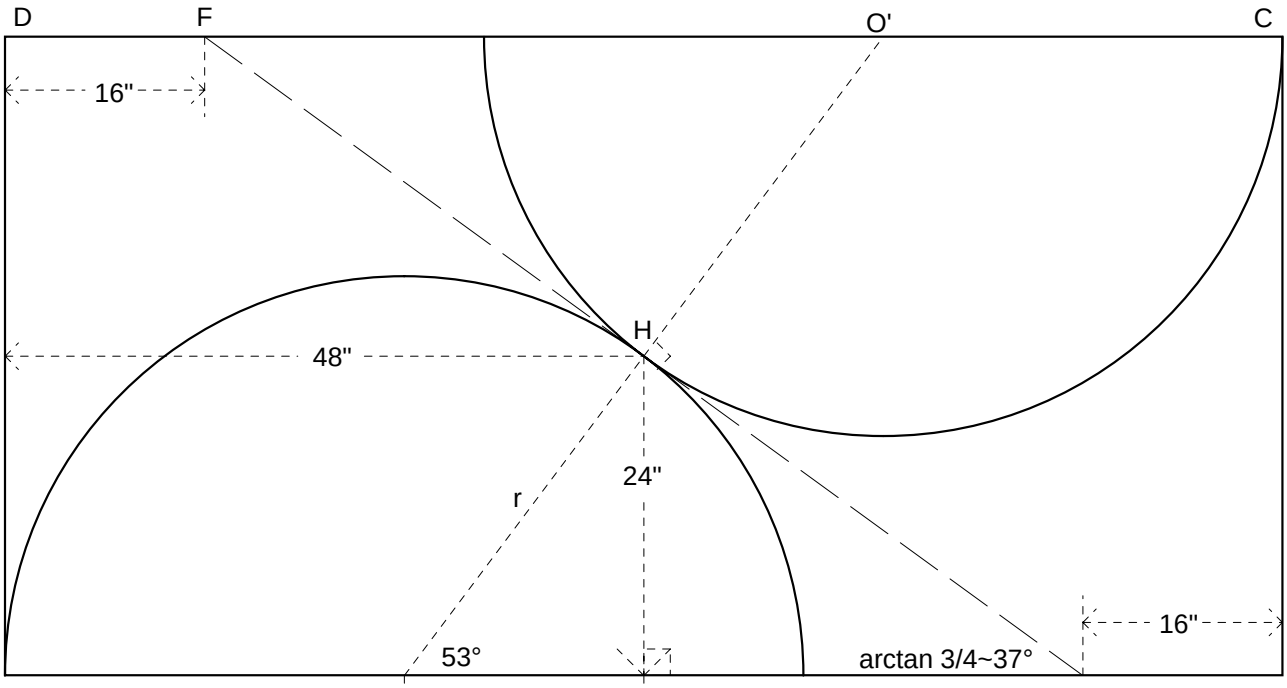

Figure 2

Approach 3: Let the cut join one corner with the opposite long side such that the length of the cut is 96" (which is the length of the sheet). Figure 3 shows that this arrangement produces a slight improvement, ~ 60.9".

Continued

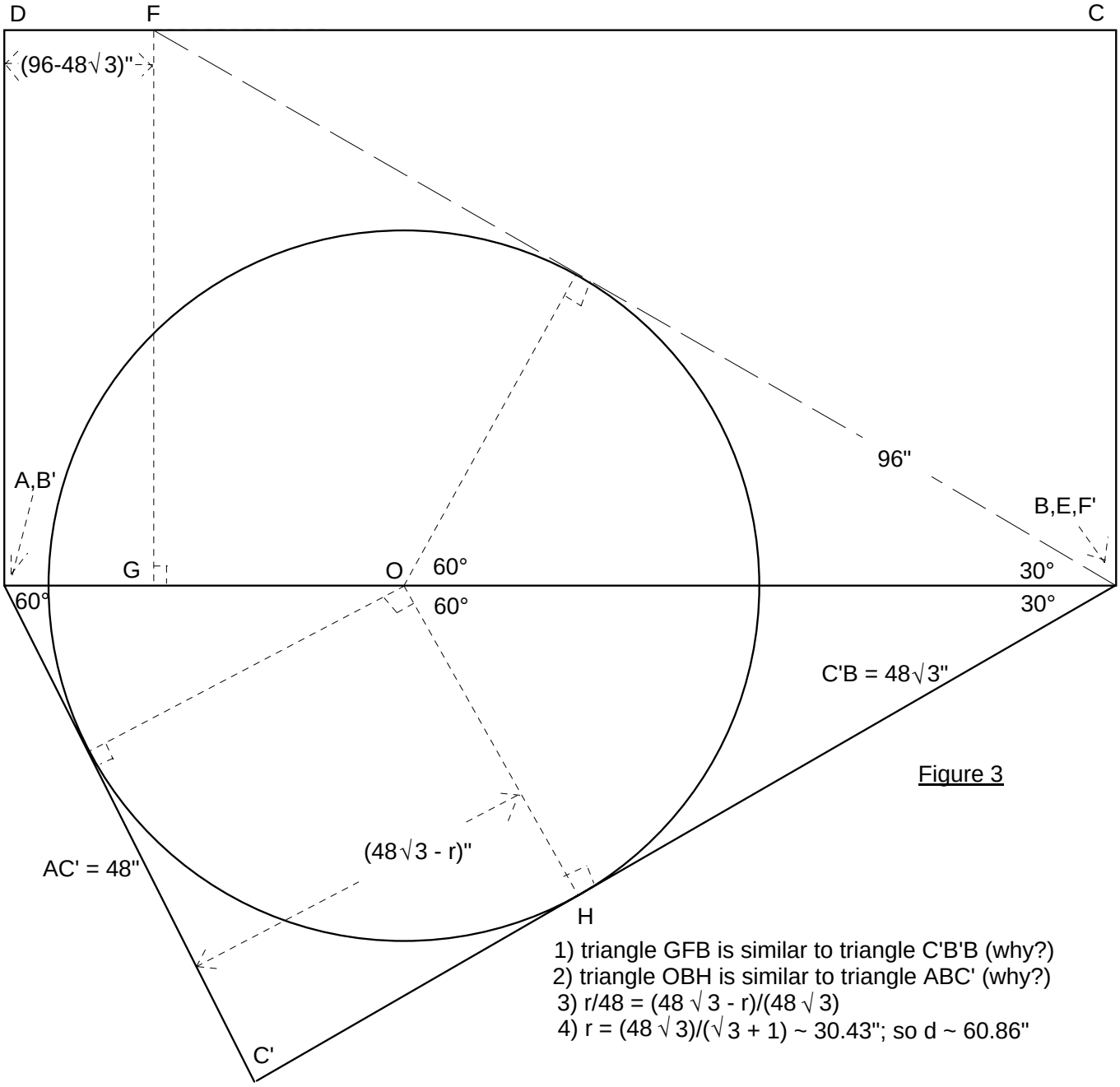

Figure 3

Approach 4: Cut an isosceles triangle from the "upper right" corner of the sheet and rejoin as shown in Figure 4. This yields a lot of 45-degree angles which make it easy to calculate the diameter as ~ 67.9". Now we're getting somewhere! What I like about this solution is that it is so elegantly simple, but works so well. But is it the best?

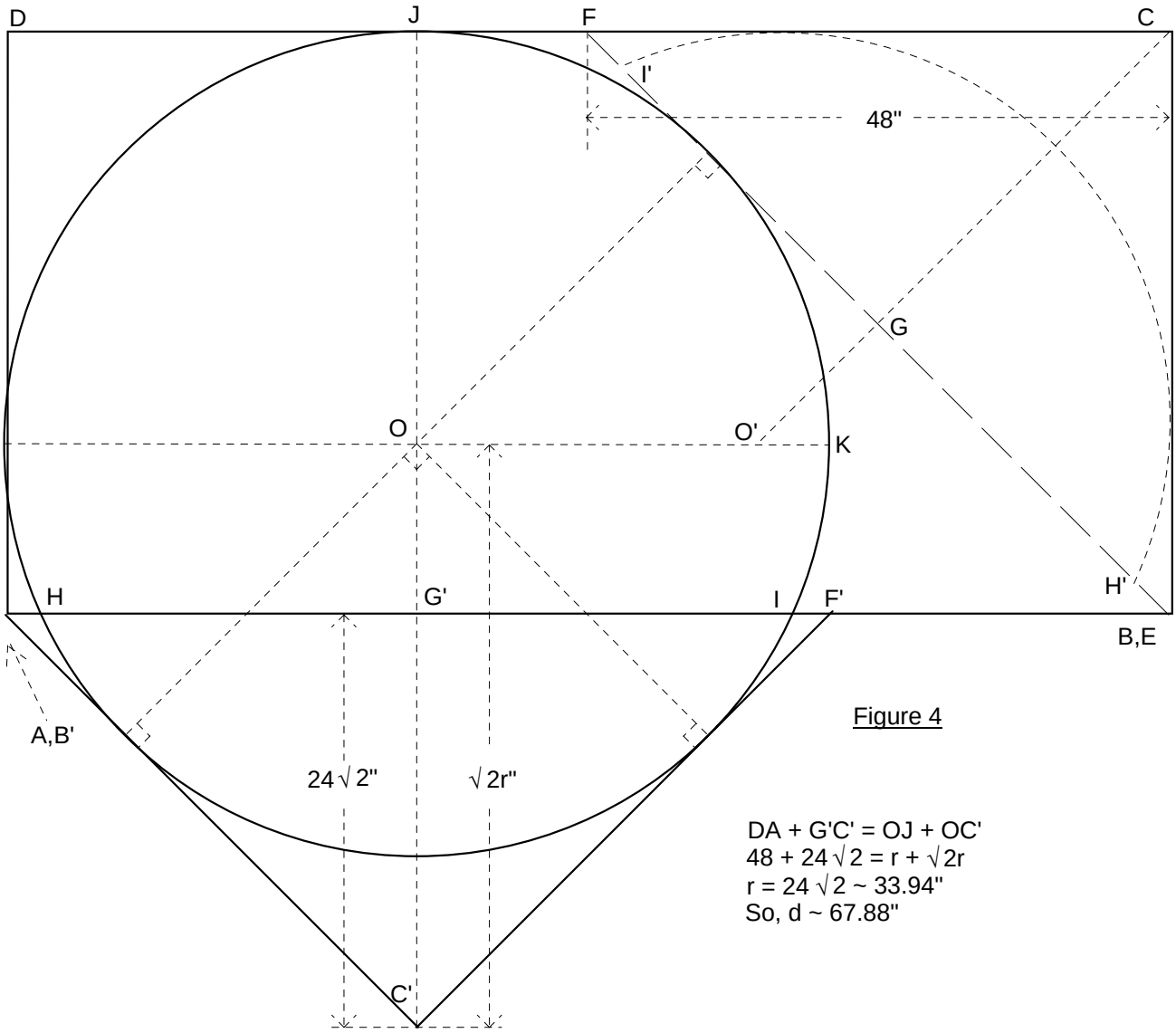

Figure 4

DA + G'C' = OJ + OC'
48 + 24√2 = r + √2r
r = 24√2 ~ 33.94"
So, d ~ 67.88"

Approach 5: Now it's your turn! Figure 5 "generalizes" Approach 4. Here are some things to consider:

1) What if a gap is left between the top of the circle and the top of the board? This means that O' may not fall on the ray OK as it did in Approach 4.
2) In Approach 4, O' lay on the bisector of angle C. Must it?
3) In Approach 4, point E coincides with a corner of the sheet. Perhaps locating it along segment AB would yield better results.
4) In Approach 4, could segment EF be rotated clockwise around point G to give an improvement? Or should point G be relocated so that it does not necessarily lie on bisector OG?

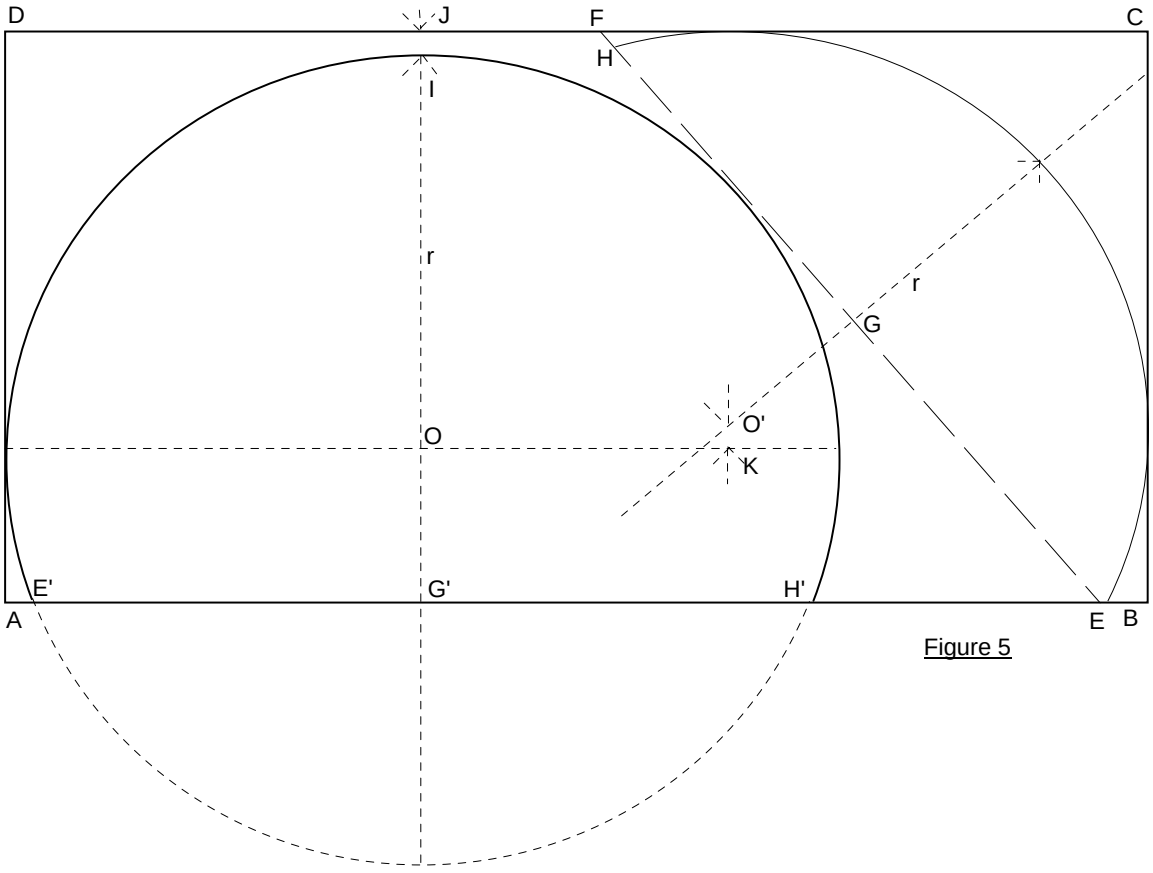

Figure 5

This is an excellent project for students to brainstorm, perhaps leading to entirely new approaches, or giving different justifications for those presented here. Experiments with scale model cardboard cutouts and guess-and-check trials would also be well suited. The mathematics involved can be kept simple, or allowed to become quite advanced. Approach 4 certainly looks elegant, but is it the best? I hope to hear of improvements!

QUOTE:
"But even in conclusions which can be known only by reasoning, I say that the testimony of many has little more value than that of few, since the number of people who reason well in complicated matters is much smaller than that of those who reason badly." Galileo Galiliei 1564-1642. As translated on page 93. *Galileo's Daughter*, 2000. Dava Sobel. Penguin Books. New York.